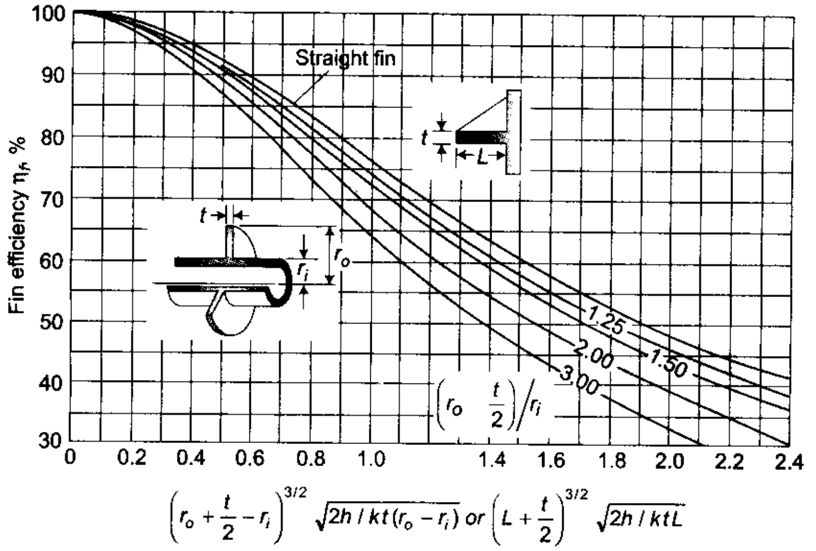

FIGURE 6.11 Efficiency of circumferential rectangular fins

Considering a fin of rectangular cross section, insulated at its end, we can write:

$$\eta_f = \frac{\tan h(m \cdot L)}{m \cdot L}$$

Now,

$$m \cdot L = \sqrt{\frac{h \cdot P}{k \cdot A_c}} \cdot L = \sqrt{\frac{h \cdot (2 \cdot w + 2 \cdot t)}{k \cdot w \cdot t}} \cdot L$$

For a very wide fin: i.e. $w >> t$, we can write:

$$m \cdot L = \sqrt{\frac{2 \cdot h \cdot w}{k \cdot w \cdot t}} \cdot L = \sqrt{\frac{2 \cdot h}{k \cdot t}} \cdot L = \sqrt{\frac{2 \cdot h}{k \cdot t \cdot L}} \cdot L^{\frac{3}{2}}$$

i.e.

$$m \cdot L = \sqrt{\frac{2 \cdot h}{k \cdot A_m}} \cdot L^{\frac{3}{2}} \quad ...(6.19)$$

where, $A_m = (L.t)$, is the profile area for the rectangular section. So, on the X-axis, instead of $(m.L)$, what is plotted is:

$$L_c^{\frac{3}{2}} \cdot \sqrt{\frac{h}{k \cdot A_m}}$$

where, L_c is the corrected length, to take into account convection from the end.

6.4.2 Fin Effectiveness (ε_f)

Consider a fin of uniform cross-sectional area A_c, fixed to a base surface. Purpose of the fin is to enhance the heat transfer. If the fin were not there, heat would have been transferred from the base area A_c, by convection. By attaching the fin, area for convection increases i.e. convective resistance ($= 1/(h.A)$) decreases; however, it is obvious that a conduction resistance due to the solid fin is now introduced and the total heat transfer would depend upon the net thermal resistance. As we go on increasing the length of fin, convection resistance will go on decreasing, but conduction resistance will go on increasing. This means that attaching a fin may not necessarily result in effectively increasing the heat transfer. Therefore, how effective the fin is in enhancing the heat transfer is characterised by a parameter called fin effectiveness.

Fin effectiveness is defined as the ratio of the heat transfer rate with the fin in place, to the heat transfer that would occur if the fin were not there, from the area of the base surface where the fin was originally fixed.

i.e. ε_f = (heat transfer rate with fin)/(heat transfer rate without fin)

i.e.
$$\varepsilon_f = \frac{Q_{fin}}{h \cdot A_c \cdot (T_o - T_a)} \quad ...(6.20)$$

Fin effectiveness equal to 1 means that there is no enhancement of heat transfer at all by using the fin; if the fin effectiveness is less than 1, that means that the fin actually reduces the heat transfer by adding additional thermal resistance! Obviously, ε_f should be as large as possible. Use of fins is hardly justified unless fin effectiveness is greater than about 2, i.e. $\varepsilon_f \geq 2$.

To get an insight into the physical implications of fin effectiveness, let us consider an infinitely long fin: Then, we have:

$$\varepsilon_f = \frac{\sqrt{h \cdot P \cdot k \cdot A_c} \cdot (T_o - T_a)}{h \cdot A_c \cdot (T_o - T_a)} \quad \textit{(fin effectiveness for very long fin)}$$

i.e.
$$\varepsilon_f = \sqrt{\frac{k \cdot P}{h \cdot A_c}} \quad ...(6.21)$$

Eq. 6.21 is an important equation. Following significant conclusions may be derived from this equation:

(i) Thermal conductivity, k should be as high as possible; that is why we see that generally, fins are made up of copper or aluminium. Of course, aluminium is the preferred material from cost and weight considerations.

(ii) Large ratio of perimeter to area of cross section is desirable; that means, thin, closely spaced fins are preferable. However, fins should not be too close as to impede the flow of fluid by convection.

(iii) Fins are justified when heat transfer coefficient h is small, i.e. generally on the gas side of a heat exchanger rather than on the liquid side. For example, the car radiator has fins on the outside of the tubes across which air flows.

(iv) Requirement that $\varepsilon_f \geq 2$, gives us the criterion:

$$\frac{k \cdot P}{h \cdot A_c} > 4 \quad ...(6.22)$$

These two important parameters, namely, η_f and ε_f are related to each other as follows:

$$\varepsilon_f = \frac{Q_{fin}}{Q_{base}} = \frac{Q_{fin}}{h \cdot A_c \cdot (T_o - T_a)} = \frac{\eta_f \cdot h \cdot A_f \cdot (T_o - T_a)}{h \cdot A_c \cdot (T_o - T_a)}$$

i.e.
$$\varepsilon_f = \frac{A_f}{A_c} \cdot \eta_f \quad ...(6.23)$$

6.4.3 *Thermal Resistance of a Fin*

Consider a fin of cross-sectional area A_c fixed on a base surface. Then, the convective thermal resistance of the base area is:

$$R_b = \frac{1}{h \cdot A_c} \quad \textit{((6.24a)...convective thermal resistance of base area)}$$

When fin is attached, we compute a thermal resistance for the fin, from the usual definition, i.e.

$$R_{fin} = \frac{\Delta T}{Q_{fin}} = \frac{T_o - T_a}{Q_{fin}} \quad \textit{((6.24b)...thermal resistance of fin)}$$

Values of Q_{fin} depend on the conditions at the tip of the fin and may be obtained from Table 6.3. Dividing equation 6.24a by Eq. 6.24b, we get:

$$\frac{R_b}{R_{fin}} = \frac{Q_{fin}}{h \cdot A_c \cdot (T_o - T_a)} = \varepsilon_f \quad \textit{((6.25)...from the definition of } \varepsilon_f \textit{ in Eqn. 6.20)}$$

Fin effectiveness may be considered as a ratio of thermal resistances and clearly, to achieve higher fin effectiveness, the conductive resistance of the fin must be smaller than the convective resistance, calculated with reference to the base cross-sectional area.

Concept of fin resistance is very useful to represent a finned surface in a thermal circuit, remembering that the conductive resistance along the fin and the convective resistance from the surface of the fin are in parallel.

6.4.4 *Total Surface Efficiency (or, overall surface efficiency, or area-weighted fin efficiency), η_t*

What we have analysed so far, is a single fin. However, in practice, a single fin is seldom used; it is always an array of fins fixed on a base surface.

In a heat exchanger, where use of fins is most prevalent, fins serve the purpose of increasing the amount of heat transferred.

Total heat exchange area (A_t) may be considered as made up of two areas:

(i) the base or prime surface area, A_p, on which there are no fins, and

(ii) the total fin surface area ($N.A_f$)

where, N is the total number of fins, and A_f is the surface area of each fin.

Now, the prime surface (or, un-finned surface) is 100% effective; but, all the fin surface area provided is not 100% effective, since there is always a temperature gradient along the fin. From the definition of fin efficiency, we know that effective area of the fin surface is $\eta_f . A_f$.

Therefore, considering the total area of the array, i.e. ($A_p + N.A_f$), we can define an total or overall surface efficiency, η_t, such that:

$$\eta_t \cdot A_t = 1 \cdot A_p + \eta_f \cdot N \cdot A_f$$

But $$A_t = A_p + N \cdot A_f$$

Therefore, $$\eta_t \cdot A_t = (A_t - N \cdot A_f) + \eta_f \cdot N \cdot A_f$$

i.e. $$\eta_t = 1 - \frac{N \cdot A_f}{A_t} \cdot (1 - \eta_f) \qquad ...(6.26)$$

Eq. 6.26 gives the value of overall or total surface efficiency (or, area –weighted fin efficiency) for a fin array. In other words, effective heat transfer area of the array is = ($\eta_t A_t$), where A_t is the total area of the prime surface plus all the fin area.

Concept of overall surface efficiency is very useful in calculating the heat transfer rates in heat exchangers where fins may be provided on one or both sides of the wall. In such a case, overall heat transfer coefficient may be obtained from:

$$U_o \cdot A_o = U_i \cdot A_i = \frac{1}{\Sigma R}$$

$$U_o \cdot A_o = U_i \cdot A_i = \frac{1}{h_o \cdot \eta_{to} \cdot A_o + R_{\text{wall}} + h_i \cdot \eta_{ti} \cdot h_i} \qquad ...(6.27)$$

where, U_o = overall heat transfer coefficient based on total outer surface area

U_i = overall heat transfer coefficient based on total inner surface area

A_o = total outer surface area

A_i = total inner surface area

η_{to} = total surface efficiency for outer surface

η_{ti} = total surface efficiency for inner surface

h_o = average heat transfer coefficient on the outer surface

h_i = average heat transfer coefficient on the inner surface

For the popular case of a tubular heat exchanger, with fins on the outside and no fins on the inside, we have:

$$\eta_{ti} = 1 \quad A_i = 2 \cdot \pi \cdot r_i \cdot L \quad \text{and,} \quad R_{\text{wall}} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L}$$

where, r_i and r_o are inside and outside radii of the tube, k is the thermal conductivity of tube material and L is the tube length.

Example 6.7. A steel rod ($k = 30$ W/(mC)), 10 mm in diameter and 50 mm long, with an insulated end is to be used as a spine. It is exposed to surroundings with a temperature of 65°C and a heat transfer coefficient of 50 W/(m^{2}C). The temperature of the base is 98°C. Determine:

(i) fin efficiency (ii) temperature at the end of spine, and (iii) heat dissipation. [M.U.]

Solution. See Fig. Example 6.7.

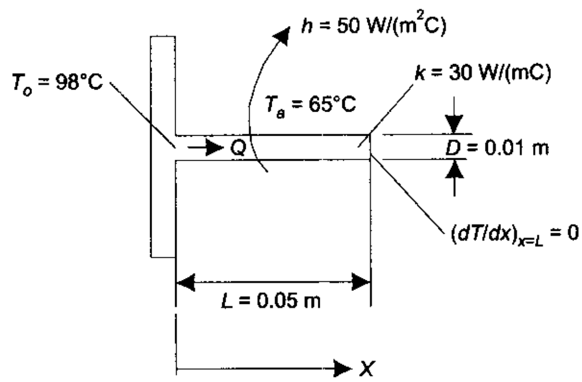

FIGURE Example 6.7 Fin of finite length, end insulated

Data:

$D := 0.01$ m $\quad L := 0.05$ m $\quad k := 30$ W/(mC) $\quad T_o := 98$°C $\quad T_a := 65$°C $\quad h := 50$ W/(m^{2}C)

Fin efficiency:

Fin efficiency for a fin with insulated end is given by Eq. 6.18:

i.e. $$\eta_f = \frac{\tan h(m \cdot L)}{m \cdot L}$$ ((6.18)...*fin efficiency for a fin with insulated end*)

First, let us calculate the parameter m:

We have: $m = \sqrt{\frac{h \cdot P}{k \cdot A_c}}$ where, P is the perimeter and A_c is the area of cross section.

Then, $$A_c := \frac{\pi \cdot D^2}{4} m^2$$ (*define the area of cross section of the rod*)

i.e. $A_c = 7.854 \times 10^{-5}$ m^2 (*area of cross section of the rod*)

and, $P := \pi \cdot D$ m (*define the perimeter of the rod*)

i.e. $P = 0.031$ m (*perimeter of the rod*)

Therefore, $$m := \sqrt{\frac{h \cdot P}{k \cdot A_c}} \text{ m}^{-1}$$ (*define the parameter m.*)

i.e. $m = 25.82$ m^{-1} (*parameter m.*)

Therefore, from Eq. 6.18:

$$\eta_f := \frac{\tan h(m \cdot L)}{m \cdot L}$$

i.e. $\eta_f = 0.666 =$ **66.6%** (*fin efficiency.*)

Temperature at the end of the spine, i.e. at $x = L$:

We use Eq. 6.7 for the temperature distribution in a fin with insulated tip:

i.e. $$\frac{T(x) - T_a}{T_o - T_a} = \frac{\cos h(m \cdot (L - x))}{\cos h(m \cdot L)} \quad ...(6.7)$$

Putting $x = L$ in Eq. 6.7, we get:

$$\frac{T_L - T_a}{T_o - T_a} = \frac{1}{\cos h(m \cdot L)}$$

i.e. $$T_L := \frac{T_o - T_a}{\cos h(m \cdot L)} + T_a$$

i.e. $T_L = 81.874°C$ ***(temperature at the end of the spine.)***

Heat dissipation from the spine:

We use Eq. 6.8 for heat dissipation from a fin with insulated end:

i.e. $$Q_{fin} = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \theta_o \cdot \tan h(m \cdot L) \quad ...(6.9)$$

Here, $\theta_o := T_o - T_a$ °C *(excess temperature at the base of fin)*

i.e. $\theta_o = 33°C$ *(excess temperature at the base of fin)*

And, $Q_{fin} := \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \theta_o \cdot \tan h(m \cdot L)$ W *(define heat transfer from the fin)*

i.e. $Q_{fin} = 1.725$ W ***(heat dissipated from the spine.***

Example 6.8. Circular aluminium fins of constant rectangular profile are attached to a tube of outside diameter $D = 5$ cm. The fins have thickness $t = 2$ mm, height $L = 15$ mm, thermal conductivity $k = 200$ W/(mC), and spacing 8 mm (i.e. 125 fins per metre length of tube). The tube surface is maintained at a uniform temperature $T_o = 180°C$, and the fins dissipate heat by convection into the ambient air at $T_a = 25°C$, with a heat transfer coefficient $h_a = 50$ W/(m^{2}C). Determine the net heat transfer per metre length of tube.

Solution. See Fig. Example 6.8.

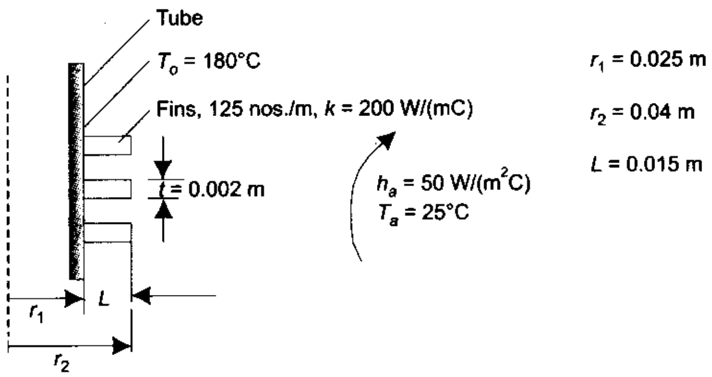

FIGURE Example 6.8 Circular fin of rectangular section

$D := 0.05$ m $L := 0.015$ m $r_1 := 0.025$ m $r_2 := 0.040$ m $t := 0.002$ m $k := 200$ W/(mC)

$T_o := 180$ C $T_a := 25$ C $h_a := 50$ W/(m^{2}C) $N := 125$

This is the case of heat transfer in a fin array. So, we will use 'total surface efficiency' concept. First, let us find out the fin parameter m: (See Table 6.4)

We have: $m = \sqrt{\frac{h_a \cdot P}{k \cdot A_c}}$ where, P is the perimeter and A_c is the area of cross section.

But, $\frac{P}{A_c} = \frac{2}{t}$ for thin fins

Therefore, $m := \sqrt{\frac{2 \cdot h_a}{k \cdot t}}$ m^{-1} *(define parameter m)*

i.e. $m = 15.811$ m^{-1} *(parameter m.)*

Fin efficiency:

Fin efficiency for circular fins is obtained from graphs in Fig. 6.11. To use those graphs, we need to calculate the following:

$$r_{2c} := r_2 + \frac{t}{2}\ \mathrm{m}$$ *(define corrected radius)*

i.e. $$r_{2c} = 0.041\ \mathrm{m}$$ *(corrected radius)*

$$L_c := L + \frac{t}{2}\ \mathrm{m}$$ *(define corrected length)*

i.e. $$L_c = 0.016\ \mathrm{m}$$ *(corrected length for fin)*

and, $$\sqrt{\frac{2 h_a}{k \cdot t \cdot (r_2 - r_1)}} = 129.1$$

Therefore, $$L_c^{3/2} \cdot \sqrt{\frac{2 h_a}{k \cdot t \cdot (r_2 - r_1)}} = 0.261$$ *(factor to be used on X-axis of Fig. 6.11)*

and, $$\frac{r_{2c}}{r_1} = 1.64$$ *(factor for use in Fig. 6.11)*

Now, with the value of 0.261 enter the X-axis of Fig. 6.11. See where the ordinate cuts the curve for $r_{2c}/r_1 = 1.64$. Move to the left and read on the Y-axis the value of η_f.

From the Fig. 6.11 we read: $\eta_f = 0.97 = 97\%$ ***(fin efficiency.)***

Alternatively:

From Table 6.4, we have:

For circular fins of rectangular section:

$$\eta_f(m, r_1, r_{2c}) = \frac{\left(\frac{2 \cdot r_1}{m}\right)}{\left(r_{2c}^2 - r_1^2\right)} \cdot \left[\frac{(k_1(m \cdot r_1) \cdot I_1(m \cdot r_{2c}) - I_1(m \cdot r_1) \cdot K_1(m \cdot r_{2c}))}{(I_0(m \cdot r_1) \cdot K_1(m \cdot r_{2c}) + K_0(m \cdot r_1) \cdot I_1(m \cdot r_{2c}))}\right]$$ *...define hf as a function of m, r_1 and r_{2c}*

In the present case, $m := 15.811 \quad r_1 := 0.025 \quad r_{2c} := 0.041$

therefore, $\eta_f(m, r_1, r_{2c}) = 0.973$ *(fin efficiency...almost same as obtained from the graph)*

(Note the ease with which Mathcad calculates the Bessel functions in the above equation.)

Total surface efficiency:

This is given by Eq. 6.26:

i.e. $$\eta_t = 1 - \frac{N \cdot A_f}{A_t} \cdot (1 - \eta_f) \quad ...(6.26)$$

where $\eta_f = 0.973$ as already calculated.

Surface area of each fin:

$$A_f := 2 \cdot \pi \cdot \left(r_{2c}^2 - r_1^2\right)\ \mathrm{m}^2$$ *(factor 2 is used to consider both upper and lower areas of the fin)*

i.e. $A_f = 6.635 \times 10^{-3}\ \mathrm{m}^2$ *(surface area of each fin)*

Prime (or base) surface area: (This is unfinned area)

$A_p := 2 \cdot \pi \cdot r_1 \cdot (1 - N \cdot t)\ \mathrm{m}^2$ *(prime surface area for 1 m length of tube)*

i.e. $A_p = 0.118\ \mathrm{m}^2$ *(prime surface area per metre length of tube)*

Therefore, total area:

$A_t := A_p + N \cdot A_f\ \mathrm{m}^2$ *(prime area plus total fin area)*

i.e. $A_t = 0.947\ \mathrm{m}^2$ *(total area)*

Applying Eq. 6.26:

$$\eta_t := 1 - \frac{N \cdot A_f}{A_t} \cdot (1 - \eta_f)$$ *(define total surface efficiency)*

i.e. $\eta_t = 0.976$ *(total surface efficiency)*

Heat transfer rate for the fin array:

Therefore, heat transfer rate is given by:

$Q := h_a \cdot \eta_t \cdot A_t \cdot (T_o - T_a)$ W/m *(define total heat transfer rate in the fin array)*

i.e. $Q = 7.167 \times 10^3$ W/m = **7.167 kW/m** ***(heat transfer rate.)***

Alternatively:

You need not memorize the Eq. 6.26 for the total surface efficiency. Just remember that prime surface is 100% effective, whereas out of the total fin area of $(N.A_f)$, only $(\eta_f . N. A_f)$ is effective. So, the total heat transfer from the fin array can be written as:

$Q := (A_p + \eta_f \cdot N \cdot A_f) \cdot h_a \cdot (T_o - T_a)$ W *(define heat transfer rate from the array)*

i.e. $Q = 7.167 \times 10^3$ W/m ***(same as obtained above.)***

Heat transfer rate if there are no fins:

It is interesting to compare the heat transfer rate obtained above, with the heat transfer rate that would be obtained **if there were no fins:**

If there are no fins, heat transfer will be by convection from the surface of the bare tube. Applying Newton's Law of Cooling, we get:

$Q_{tube} := h_a \cdot (2 \cdot \pi \cdot r_1 \cdot 1) \cdot (T_o - T_a)$ W *(define heat transfer by convection from tube surface)*

i.e. $Q_{tube} = 1.217 \times 10^3$ W *(heat transfer rate from the bare tube)*

We get: $\dfrac{Q}{Q_{tube}} = 5.887$

i.e. heat transfer increases by nearly 6 times because of addition of fins.

6.5 Application of Fin Theory for Error Estimation in Temperature Measurement

Temperature of a fluid flowing in a pipe is generally measured with a thermometer placed in a thermowell welded radially or obliquely to the pipe wall. Thermowell is a thin tube, generally of a material of low thermal conductivity, such as stainless steel, filled with oil, for better thermal contact with the thermometer bulb. See Fig. 6.12.

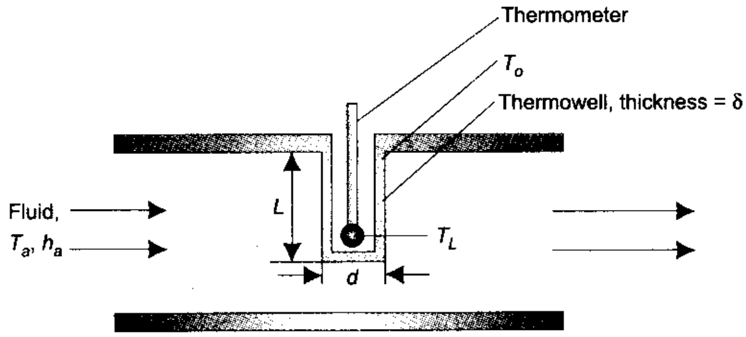

FIGURE 6.12 Error estimation in temperature measurement

Let L = length of thermowell

d = diameter of thermowell

δ = thickness of thermowell wall

T_o = temperature at the root of thermowell, i.e. on the pipe surface

T_a = temperature of the fluid flowing, and

T_L = temperature measured by the thermometer

h_a = heat transfer coefficient between the thermowell and the fluid.

If the temperature of a hot fluid flowing in the pipe is T_a, obviously, the temperature indicated by the thermometer, T_L will not be equal to T_a, but less than T_a, because of heat loss along the wall of the thermowell from its tip to the root (and, vice versa, for a cold fluid flowing in the pipe).

Our aim is to estimate the error in the thermometer reading.

We apply the fin theory. Considering the thermowell to be a fin protruding from the pipe wall, with an insulated tip (i.e. no heat transfer from its tip), we can write, from Eq. 6.7:

$$\frac{T(x) - T_a}{T_o - T_a} = \frac{\cos h(m \cdot (L - x))}{\cos h(m \cdot L)} \quad \text{...(6.7)}$$

At the tip, i.e. at $x = L$, $T(x) = T_L$.

Substituting $x = L$ in Eq. 6.7:

$$\frac{T_L - T_a}{T_o - T_a} = \frac{1}{\cos h(m \cdot L)} \quad \text{...(a)}$$

And, the error in thermometer reading is given by:

$$T_L - T_a = \frac{T_o - T_a}{\cos h(m \cdot L)} \quad \text{...(b)}$$

From Eq. b, we observe that to reduce the temperature error, we should have the factor $1/\cos h(m.L)$ as small as possible. To achieve this, looking at the graph of $1/\cos h(m.L)$ vs. $(m.L)$ given in section 6.2.2, it is clear that

$$\sqrt{\frac{h \cdot P}{k \cdot A_c}} \cdot L$$

must be as large as possible.

This leads to the following conclusions:

(i) value of heat transfer coefficient, h should be large

(ii) value of thermal conductivity, k should be small

(iii) thermowell should be long and thin-walled. (thermowell may be placed obliquely inside the pipe, to make it long).

Again, for the thermowell, treated as a fin, we have:

$$m = \sqrt{\frac{h \cdot P}{k \cdot A_c}}$$

and,

$$\frac{P}{A_c} = \frac{\pi \cdot d}{\pi \cdot d \cdot \delta} \quad \text{for } \delta << d$$

i.e.

$$\frac{P}{A_c} = \frac{1}{\delta}$$

i.e. fin parameter, m does not depend upon thermowell pocket diameter, when the wall thickness is very small compared to its diameter.

Example 6.9. The temperature of air in an air stream in a tube is measured by a thermometer placed in a protective well filled with oil. The thermowell is made of steel tube of 1.5 mm thick sheet of length 120 mm. The thermal conductivity of steel = 58.8 W/(mK). and h_a = 23.3 W/(m^{2}K). If the air temperature recorded was 84°C, estimate the measurement error, if the temperature at the base of the well was 40°C. [M.U.]

Solution. See Fig. Example 6.9.

Data:

$L := 0.12$ m $\quad \delta := 0.0015$ m $\quad k := 58.8$ W/(mK) $\quad T_o := 40$°C $\quad T_L := 84$°C $\quad h_a := 23.3$ W/(m^{2}C)

Let T_a be the temperature of air flowing.

Let us first calculate fin parameter m:

$$m = \sqrt{\frac{h_a \cdot P}{k \cdot A_c}}$$

Again, $\frac{P}{A_c} = \frac{\pi \cdot d}{\pi \cdot d \cdot \delta} = \frac{1}{\delta}$ *(where, P is the perimeter, d is the diameter of thermowell.)*

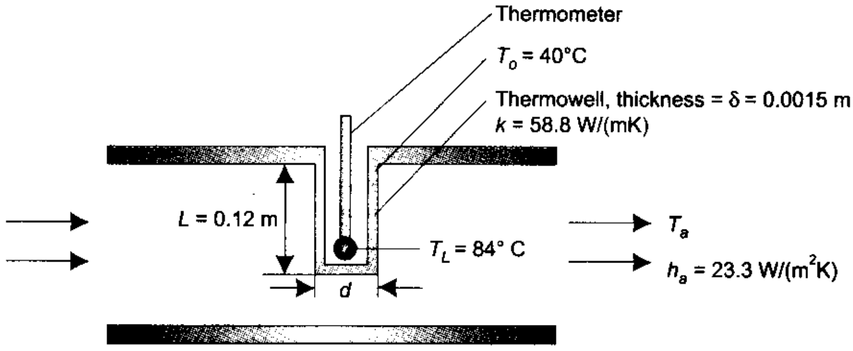

FIGURE Example 6.9 Error estimation in temperature measurement

i.e. $$m := \sqrt{\frac{h_a}{k \cdot \delta}}$$

or, $m = 16.253 \text{ m}^{-1}$ *(fin parameter m)*

and, $\cos h(m \cdot L) = 3.5869$

Considering the thermowell as a fin with insulated end, we have:

$$\frac{T_L - T_a}{T_o - T_a} = \frac{1}{\cos h(m \cdot L)}$$

i.e. $$\frac{84 - T_a}{40 - T_a} = \frac{1}{3.5869}$$

or, $$T_a := \frac{(3.5869 \times 84) - 40}{2.5869}$$

i.e. $T_a = 101.009°\text{C}$ *(temperature of air flowing.)*

Error in temperature measurement:

Actual temperature of air is 101.009°C, while recorded temperature is 84°C.

i.e. $T_a - T_L = 17.009°\text{C}$ *(error in temperature measurement)*

or, $$\frac{T_a - T_L}{T_a} \cdot 100 = 16.839$$ *(Percentage error in measurement of temperature = 16.8%.)*

6.6 Summary

Fins are widely used in industry to enhance heat transfer from surfaces. In this chapter, first, we derived the general differential equation governing the temperature distribution in a fin, from an energy balance on a differential element of the fin. Subsequently, solution of this differential equation with different boundary conditions was obtained to get temperature distribution in the fin. Once the temperature distribution is known, heat transfer rate through the fin is easily calculated by applying Fourier's law. Four important cases considered were:

Case (i): Infinitely long fin,

Case (ii): Fin insulated at its end (i.e. negligible heat loss from the end of the fin),

Case (iii): Fin losing heat from its end by convection, and

Case (iv): Fin with specified temperature at its two ends.

Performance of fins was discussed with reference to parameters such as fin efficiency and fin effectiveness; concepts of thermal resistance of fins and total surface efficiency, or, area weighted fin efficiency of a fin array was explained.

Graphs and tables for practically important fin geometries were presented.

Finally, application of fin theory to correction of error in temperature measurement was studied.

In this text, in our study of heat conduction so far, one of the assumptions in our analysis was steady state heat conduction, i.e. temperature at any given location in the solid was assumed to be constant and did not change with time. In the next chapter, we shall study the cases of transient conduction, i.e. when the temperature at a given location in the solid changes with time.

Questions

1. Explain why fins are widely used. Discuss a few commonly used types of fins.
2. 'Addition of fins may not necessarily increase the heat transfer from a surface; it may even decrease the heat transfer'—comment on this statement.
3. Define 'fin efficiency' and 'fin effectiveness'. Explain, as a corollary, why thin, closely spaced fins of a material of good thermal conductivity are preferable.
4. Explain why fins are generally used on the gas side in a gas-to-liquid heat exchanger.
5. For an infinitely long fin, with usual notations, prove that heat dissipated is given by:
$$Q_{\text{fin}} = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \theta_o = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot (T_o - T_a)$$
6. Using usual notations and starting from basics, derive and solve a differential equation for heat flow through a moderately long pin fin (dT/dx at $x = L$ is zero) to get an expression for the non-dimensional temperature distribution along the length of the fin as:
$$\frac{\theta(x)}{\theta_o} = \frac{\cos h(m \cdot (L - x))}{\cos h(m \cdot L)}$$
and also show that heat transferred in the fin is given by:
$$Q_{\text{fin}} = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \theta_o \cdot \tan h\,(m \cdot L).$$
7. A thin fin of length L, has its two ends attached to two parallel walls which have temperatures, T_1 and T_2. The fin loses heat by convection to ambient air at T_∞. Obtain an analytical expression for the one dimensional temperature distribution along the length of the fin.
8. The end of a very long cylindrical rod is attached to a heated wall and its surface is in contact with a cold fluid. If the rod diameter were doubled, by what percentage would the heat transfer rate change?

Problems

1. A copper pin fin, 0.25 cm diameter, protrudes from a wall at 95°C into ambient air at 25°C. The heat transfer is mainly by free convection with heat transfer coefficient, h = 10 W/(m^{2}K). Calculate the heat loss assuming that the fin is infinitely long. For copper, take k = 395 W/(mK).
2. Calculate the rate of heat loss from a rectangular fin of length 2 cm, on a plane wall. Thickness of fin is 2 mm and its breadth is 20 cm. Take θ_1 = 200°C, h = 17.5 W/(m^{2}K), k = 52.5 W/(mK). Assume that heat loss from the tip is negligible.
3. Aluminum square fins (0.5 mm x 0.5 mm) of 10 mm length are provided on the surface of an electronic device to carry 45 mW of energy generated by the device. The temperature at the surface of the device should not exceed 80°C, while temperature of the surrounding medium is 40°C. Assume k for aluminium = 190 W/(mK), h = 12W/(m^{2}K). Find the number of fins required, neglecting heat loss from the end of the fin.
4. An aluminum fin, 0.5 mm square and 1cm long, is attached to a semiconductor device to provide additional cooling. The base of the fin can be assumed to be at the inside temperature of 80°C. Find the cooling capacity provided by the fin. Ambient temperature = 40°C, k = 177 W/(mK), h = 12.44 W/(m^{2}K).
5. One end of a copper rod, 15 cm long and 0.6 cm diameter, is connected to a wall at 200°C while the other end protrudes into a room whose air temperature is 21°C. If the tip of the rod is insulated, estimate the heat lost by the rod, assuming the heat transfer coefficient between its surface and surrounding air as 28 W/(m^{2}K). Also, calculate the efficiency of the fin. Take k for copper = 370 W/(mK). State the assumptions made.
6. A cylinder 5 cm diameter and 1 m long, is provided with 12 longitudinal, straight fins of 1 mm thick and 2.5 cm height. k of fin material is 75 W/(mK). Calculate the heat lost from the cylinder if the surface temperature of the cylinder is 200°C and that of the surrounding is 40°C. Given: heat transfer coefficient between the cylinder and fins and surrounding air = 25 W/(m^{2}K).
7. Circumferential fins of constant thickness of 1 mm (k = 190 W/(mK)), are attached on a 50 mm OD pipe at a pitch of 5 mm. Fin length is 20 mm. Wall temperature is 150°C. Convection heat transfer coefficient is 45 W/(m^{2}K). Determine heat flow rate from 1 m length of pipe. Compare the heat flow with fins to that without the fins.

8. A hot plate (1 m × 1 m), at 150°C is to be cooled by attaching on its surface, 10,000 number of cylindrical, pin fins of each, 3 mm diameter and 3 cm long. Surrounding air is at 25°C. Heat transfer coefficient between the fin surfaces and the surroundings is 30 W/(m^{2}C). Determine:
   (i) overall surface effectiveness
   (ii) heat transfer rate, with the fins in place
   (iii) heat transfer rate from the plate, if there were no fins
   (iv) decrease in thermal resistance due to attaching the fins.
9. An aluminium fins are fixed on one side (size: 1 m x 1 m), of an electronic device to increase the heat dissipation. Fins are of rectangular cross section, 0.2 cm thick and 3 cm long. There are 100 fins per metre. Convection heat transfer coefficient for both the plate and the fins is 30 W/(m^{2}K). Determine the percentage increase in the rate of heat transfer due to attaching the fins.
10. An iron bar, 15 mm in diameter, spans the distance between two plates, 50 cm apart. Air at 25°C flows in the space between the plates resulting in heat transfer coefficient of 15 W/(m^{2}K). Calculate the heat transfer and temperature at the middle of the bar, if the plates are maintained at 125°C each. For iron, k = 45 W/(mK).
11. Two ends of a 6 mm diameter copper rod (U-shaped) having k = 330 W/(mK), are rigidly connected to a vertical wall as shown in Fig. Problem 6.11. Wall temperature is constant at 100°C. Developed length of the rod is 50 cm and is exposed to air at 30°C. Combined convective and radiative heat transfer coefficient is 30 W/(m^2 K). Calculate:
   (i) the temperature at the centre of the rod
   (ii) net heat transfer from the rod to air.

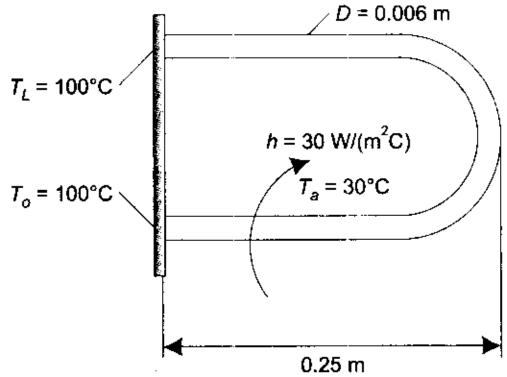

FIGURE Problem 6.11 U-shaped rod, both ends fixed to a wall

12. A steel rod (k = 55 W/(mK)), of length 50 cm, diameter 2.5 cm, has its two ends maintained at 150°C and 60°C. Ambient air, to which heat is dissipated by the rod, is at 25°C and the heat transfer coefficient is 20 W/(m^2 K). Determine:
   (i) minimum temperature in the rod
   (ii) temperature at the mid-point of the rod, and
   (iii) heat transfer rates from the left and right ends.
13. A Hg-thermometer placed in a well filled with oil, is required to measure the temperature of compressed air flowing in a pipe. The well is 14 cm long and is made of steel 1.5 mm thick. The temperature indicated by the thermometer is 100°C. The pipe wall temperature is 50°C. The film coefficient outside the wall is 30W/(m^{2}C). Estimate the % error in measurement of temperature of air. k for steel = 40W/(mC).

CHAPTER

7

Transient Heat Conduction

7.1 Introduction

In chapter 3, we derived the general differential equation for conduction and then applied it to problems of increasing complexity, e.g. first, we studied heat transfer in simple geometries without heat generation and then we studied heat transfer when there was internal heat generation. In all these problems, steady state heat transfer was assumed, i.e. the temperature within the solid was only a function of position and did not depend on time, i.e. mathematically, $T = T(x, y, z)$. However, all the process equipments used in engineering practice, such as boilers, heat exchangers, regenerators, etc. have to pass through an unsteady state in the beginning when the process is started, and, they reach a steady state after sufficient time has elapsed. Or, as another example, a billet being quenched in an oil bath, goes through temperature variations with both position and time before it attains a steady state. Conduction heat transfer in such an unsteady state is known as transient heat conduction or, unsteady state conduction, or time dependent conduction. Obviously, in transient conduction, temperature depends not only on position in the solid, but also on time. So, mathematically, this can be written as $T = T(x, y, z, \tau)$, where τ represents the time coordinate.

Naturally, solutions for transient conduction problems are a little more complicated compared to steady state analysis, since now, an additional parameter, namely time (τ) is involved.

Typical examples of transient conduction occur in:

(i) heat exchangers
(ii) boiler tubes
(iii) cooling of cylinder heads in I.C. engines
(iv) heat treatment of engineering components and quenching of ingots
(v) heating of electric irons
(vi) heating and cooling of buildings
(vii) freezing of foods, etc.

Two types of transient conduction may be identified:

(a) periodic heat flow problems, where the temperatures vary on a regular, periodic basis, e.g. in I.C. engine cylinders, alternate heating and cooling of earth during a 24 hr cycle (by sun) etc.
(b) non-periodic heat flow problems, where temperature varies in a non-linear manner with time.

To solve a given one-dimensional, transient conduction problem, one could start with one of the relevant general differential equations discussed in chapter 3 and by solving it in conjunction with appropriate boundary conditions, and get the temperature distribution as a function of position and time. For example, for one-dimensional conduction, in Cartesian coordinates, we have:

$$\frac{d^2T}{dx^2} = \frac{1}{\alpha} \cdot \frac{dT}{d\tau}$$ *-without heat generation)*